Oxford Mathematics Alphabet

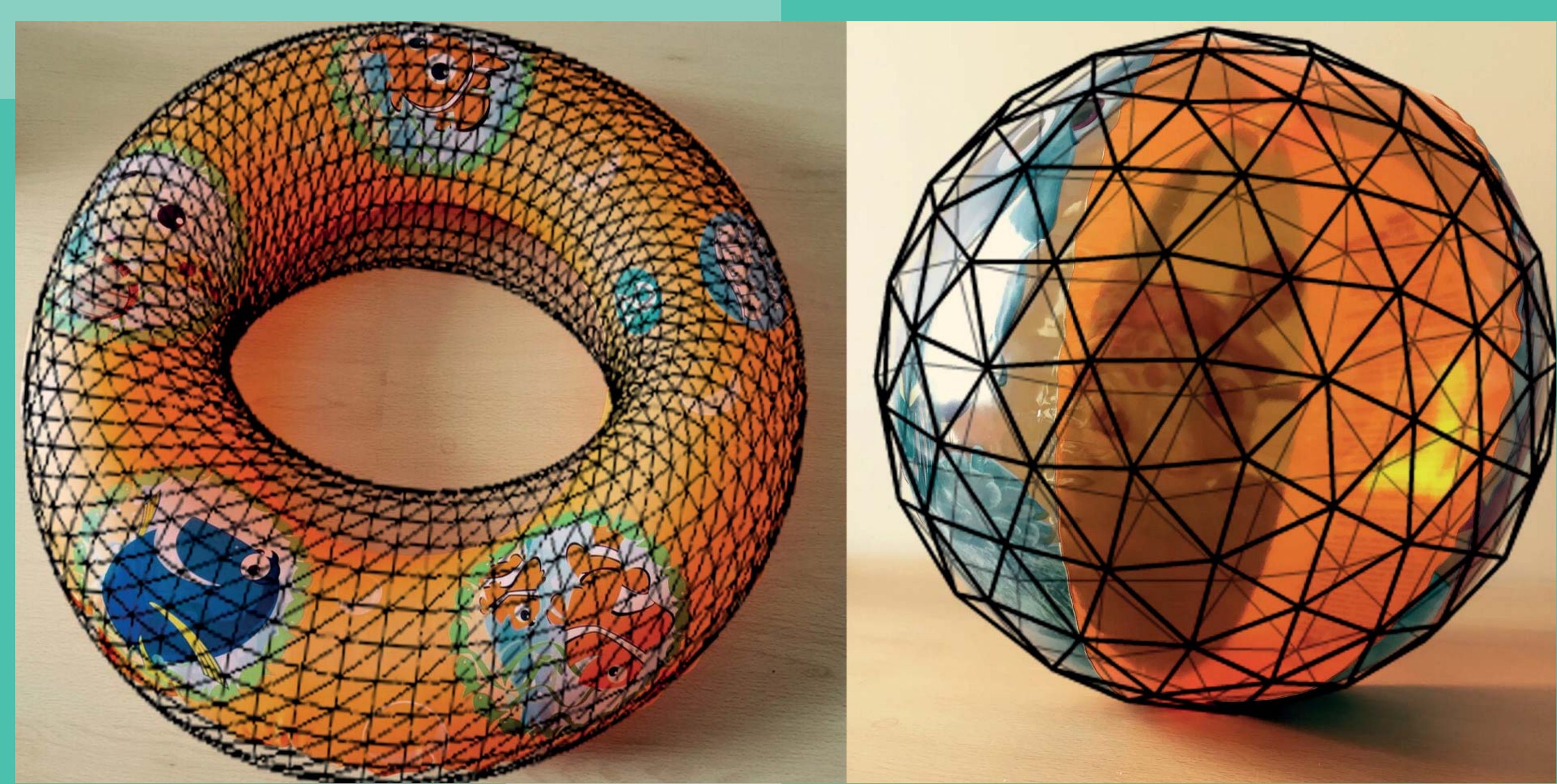
H *is for* homology

A life belt, a coffee cup, a jumping ball, a beach ball – what do these objects have in common? What sets them apart? It is questions like these that are considered in the mathematical field called *topology*. A method to study these questions is given by the theory of homology.

Homology is a mathematical way of counting different types of loops and holes in topological spaces. A life belt has two different loops. One surrounds the hole in the middle which can be occupied by a swimming person. The second loop surrounds the inflated inside of the life belt. Similarly, a (hollow) coffee cup has two different kinds of loops on its surface. Every loop on a beach ball can be contracted to a single point. However, if we cut the beach ball open, we find a hollow inside, i.e. a higher dimensional hole.

You can only change the number of voids/holes and components of an object if you cut and glue. Conversely, by bending and stretching the number of holes/voids or components does not change. Homology is therefore a so-called *topological invariant*.

Mathematically, homology associates a number of algebraic objects to topological spaces such as beach balls and life belts. There are different types of homology theories. For example, in simplicial homology, one simplifies topological spaces by using building blocks such as points, edges between points, triangles (consisting of three points, three edges and a face), tetrahedra etc., which are called simplices. One can combine these building blocks into scaffolds of the topological space, called simplicial complexes.



The applications of homology are manifold(s). For example, within mathematics one can prove that there is no possibility to deform a two-dimensional space into a three-dimensional space.

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